symmetry the wave profile (curve *1*) differs from the plot showing the dependence of the pressure at the piston on time (curve *2*) by 15–20% on average. The law governing the drop in pressure in a cavity differs considerably in the axisymmetric case from the exponential relation (4.5).

The author thanks A.L. Gonor for discussing the problem.

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PMM U.S.S.R.,Vol.49,No.2,pp.190-195,1985
Printed in Great Britain

0021-8928/85 $10.00+0.00
Pergamon Journals Ltd.

DIFFRACTION OF A SINGLE PLANE WAVE BY A V-SHAPED WING*

P.V. TRET'YAKOV

A linear formulation is used to solve the problem of the diffraction of a single plane wave by a *V*-shaped wing moving at supersonic speed. The solution is based on the study of the eigenfunctions for a class of selfsimilar solutions of the three-dimensional wave equation. The boundary integral is constructed using a method analogous to that discussed in /1/, and results obtained in /1–3/ are used.

1. We shall seek a solution of the wave equation

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}-\frac{\partial^2\Phi}{\partial t^2}=0 \tag{1.1}$$

for the homogeneous functions of zero dimensions in t and $q=(x^2+y^2+z^2)^{1/2}$.

It was shown in /1/ that knowing the homogeneous solution of zero dimensions and using the relation

$$\Phi_n=\frac{(-1)^n(t^2-q^2)^{n+1}}{2^n n!}\frac{\partial^n}{\partial t^n}\left(\frac{\Phi_0}{t^2-q^2}\right)$$

we can obtain a uniform solution of dimensions $n\in N$. Here Φ_0 and Φ_n are solutions of (1.1) uniform in t and q, of dimensions zero and n respectively.

We have the following representation for the uniform solution Φ_0 of zero dimensions in the form of a series in eigenfunctions:

$$\Phi_0=-(\rho^2-1)\frac{\partial}{\partial\rho}\sum_{n=0}^{\infty}\Big[\frac{A_{n,0}}{2}G_{n,0}(\varphi)Q_n(\rho)+$$

$$\sum_{k=1}^{\infty}(A_{n,k}\cos k\lambda\theta+B_{n,k}\sin k\lambda\theta)G_{n,k}(\varphi)Q_{n+k\lambda}(\rho)\Big]$$

$$G_{n,k}^{(\varphi)}=C_n^{k\lambda+1/2}(\cos\varphi)\sin^{k\lambda}\varphi$$

$$\left.\begin{matrix}A_{n,k}\\B_{n,k}\end{matrix}\right\}=\frac{2n!\,(n+k\lambda+1/2)\,\Gamma(k\lambda+1/2)\,\Gamma(2k\lambda+1)}{T\sqrt{\pi}\,\Gamma(n+2k\lambda+1)\,\Gamma(k\lambda+1)}\times$$

*Prikl.Matem.Mekhan.,49,2,251-257,1985

$$\int_0^T \int_0^\pi G_{n,k}(\chi) F(\psi, \chi) \sin\chi \, d\chi \begin{Bmatrix} \cos k\lambda\psi \\ \sin k\lambda\psi \end{Bmatrix} d\psi$$

$$F(\theta, \varphi) = \Phi_0|_{t=q}, \quad \lambda = \frac{2\pi}{T}, \quad \rho = \frac{t}{q}.$$

Here θ and φ are spherical angular coordinates, $C_n^\nu(\cos\varphi)$ are the Gegenbauer polynomials, $Q_\mu(\rho)$ is the Legendre function of second kind, $\Gamma(x)$ is the gamma function and T is the period of the solution in θ.

Summing the series obtained over n (see /2/) and taking into account the relation

$$\frac{1}{2} + \sum_{k=1}^{\infty} \cos k\lambda(\psi - \theta)\xi^{k\lambda} = \frac{1 - \xi^{2\lambda}}{1 - 2\xi^\lambda \cos\lambda(\psi - \theta) + \xi^{2\lambda}}, \quad |\xi| < 1$$

we obtain

$$\Phi_0 = -(\rho^2 - 1)\frac{\partial}{\partial\rho}\frac{1}{T}\int_0^T\int_0^\pi \frac{F(\psi, \chi)(1 - \xi^{2\lambda})\sin\chi \, d\chi \, d\psi}{\Omega[1 - 2\xi^\lambda\cos\lambda(\psi - \theta) + \xi^{2\lambda}]} \tag{1.2}$$

$$\Omega = [(\rho - \cos\varphi\cos\chi)^2 - \sin^2\varphi\sin^2\chi]^{1/2}$$

$$\xi = \frac{\sin\varphi\sin\chi}{\Omega + \rho - \cos\varphi\cos\chi}.$$

This boundary integral is identical, when $T = 2\pi$ and $T = 4\pi$, with the boundary integrals obtained earlier /1, 4/.

2. Using the integral bobtained, we shall solve the problem of diffraction of a single plane wave by a V-shaped wing moving with constant supersonic velocity (with supersonic edges).

Let the V-shaped wing with sweepback angle at the tip of $\pi/2 - \nu$ and V-shaped angle $\pi - 2\beta$ (Fig.1) move symmetrically in the negative z direction of a Cartesian coordinate system with constant supersonic velocity ($M > 1$). The y axis lies in the plane of symmetry of the wing. We choose the angular spherical coordinates as follows:

$$\theta = \operatorname{arc\,tg}\frac{y}{x}, \quad \varphi = \operatorname{arc\,tg}\frac{\sqrt{x^2 - y^2}}{z}.$$

Let a single plane wave impinge on the wing ($H(x)$ is the Heaviside function)

$$\Phi = H(t - q\cos\varphi\cos\alpha + q\sin\varphi\sin\theta\sin\alpha), \ \alpha = \text{const}.$$

Fig.2 shows the diffraction pattern. The flow will be three-dimensional only within the diffracted sphere with centre at the point O, in the remaining regions the solution can be obtained with help of the equations describing plane motion.

In the regions $AECQ$, $AE'BQ'$, $CNGMH$, $BNGM'H'$ and $NMGM'T$ and within the cones with apices at the points A, B, C (henceforth we shall denote the cones with apices at the corresponding points by those letters, e.g. cone A) the solution is obtained in the same manner as in /1/

$$\Phi = 1 + L, \quad L = \frac{M\sin\nu\cos\beta}{\sqrt{M^2\sin^2\nu - 1}} \quad (AECQ, AE'BQ')$$

$$\Phi = 2 \ (CNGMH, BNGM'H'), \ \Phi = 3 \ (NMGM'T).$$

To ensure that the solution within the cones depends on three variables (x_1, y_1, τ) only and diffraction pattern within the cones reduces to the form shown in Fig.3, we must carry out the following coordinate transformation:

$$\begin{aligned} x_1 &= x\cos\beta\cos\mu \pm y\sin\beta\cos\mu \mp z\sin\mu \\ y_1 &= y\cos\beta \mp x\sin\beta \\ z_1 &= \pm x\sin\mu\cos\beta - y\sin\mu\sin\beta - z\cos\mu \\ \tau &= (\zeta_0 t - z_1)(\zeta_0^2 - 1)^{-1/2}, \ \cos\mu = \zeta_1(\zeta_0\cos\gamma)^{-1} \\ \zeta_0 &= [(M\cos\alpha - 1)^2\sin^2\nu + \zeta_1^2]^{1/2}(\cos\gamma)^{-1} \\ \zeta_1 &= M\sin\nu\sin\alpha\cos\beta - \cos\nu, \quad \cos\gamma = \cos\alpha\cos\nu - \\ &\quad \sin\alpha\sin\beta\sin\nu \end{aligned} \tag{2.1}$$

where the upper and lower signs correspond to cones B and C.

For cone A we have

$$x_1 = x, \quad y_1 = y, \quad z_1 = -z, \quad \tau = \frac{Mt - z_1}{\sqrt{M^2 - 1}}.$$

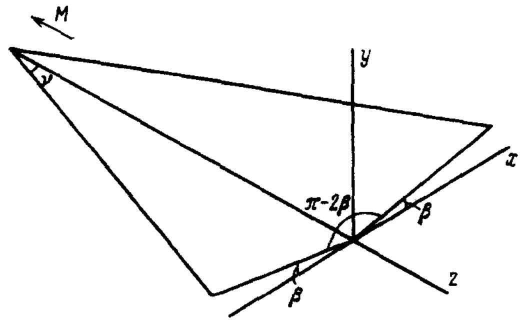


Fig.1

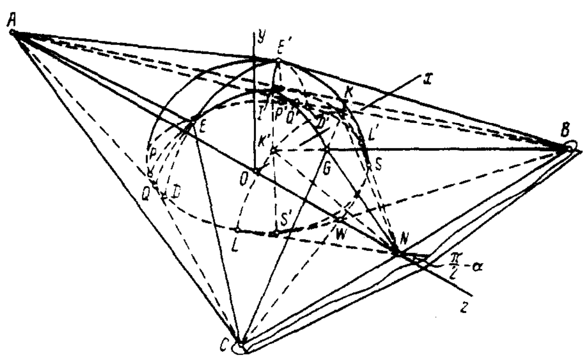


Fig.2

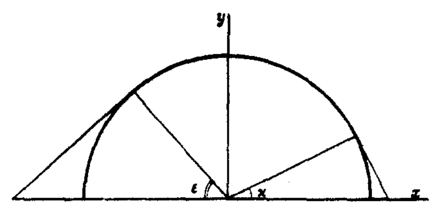


Fig.3

Moreover, to reduce the diffraction pattern in this cone to the form shown in Fig.3, we must carry out the conformal transformation

$$R_2 \exp(i\theta_2) = R_1^{\lambda} \exp(i\lambda(\theta_1 - \beta))$$

$$R_1 = \frac{\tau}{r_1} - \sqrt{\frac{\tau^2}{r_1^2} - 1}, \quad r_1 = \sqrt{x_1^2 - y_1^2}, \quad \theta_1 = \operatorname{arc\,tg} \frac{y_1}{x_1},$$

$$\lambda = \frac{\pi}{\pi - 2\beta} \quad R_2 = \frac{\tau}{r_2} - \sqrt{\frac{\tau^2}{r_2^2} - 1}, \quad r_2 = \sqrt{x_2^2 - y_2^2},$$

$$\theta_2 = \operatorname{arc\,tg} \frac{y_2}{x_2}.$$

The boundary conditions at the cone surfaces and the angles separating various boundary conditions (Fig.3) will be as follows:

$$0 \leqslant \theta_1 \leqslant \varkappa_B \Phi = 1 + L; \quad \varkappa_B < \theta_1 < \pi - \varepsilon_B \Phi = 1, \quad \pi - \varepsilon_B \leqslant \theta_1 \leqslant \pi \Phi = 2 \qquad (B)$$

$$\sin \varepsilon_B = \frac{\zeta_0 \sin\alpha \cos\beta}{\sqrt{\zeta_0^2 - 1}}, \quad \sin \varkappa_B = \frac{\zeta_0 \sin\alpha \cos\beta}{L\sqrt{\zeta_0^2 - 1}}$$

$$0 \leqslant \theta_1 \leqslant \varkappa_C \Phi = 2, \quad \varkappa_C < \theta_1 < \pi - \varepsilon_C \Phi = 1, \quad \pi - \varepsilon_C \leqslant \theta_1 \leqslant \pi \Phi = 1 - L \qquad (C)$$

$$\varepsilon_C = \varkappa_B, \quad \varkappa_C = \varepsilon_B$$

$$0 \leqslant \theta_2 \leqslant \lambda\varkappa \text{ and } \pi - \lambda\varepsilon \leqslant \theta_2 \leqslant \pi \Phi = 1 - L,$$

$$\lambda\varkappa < \theta_2 < \pi - \lambda\varepsilon \Phi = 1 \qquad (A)$$

$$\sin\varepsilon = \sin\varkappa = \frac{M \sin\alpha \cos\beta}{L\sqrt{M^2 - 1}}$$

where the letters in parenthesis denote the corresponding cones. We have the condition of impermeability $\partial\Phi / \partial n = 0$ at the wing surface for all cases. In the new coordinates this condition becomes $\partial\Phi / \partial y_1|_{y_1=0} = 0$, and we can therefore continue symmetrically onto the half-plane $y_1 < 0$ the boundary conditions for each cone and use Poisson's integral. Since the solution is obtained in the same manner for all cases, we give here the solution for cone C only

$$\Phi = 1 + Z(\varkappa - \theta_1) - Z(\varkappa + \theta_1) - L\,[Z(\pi + \varepsilon - \theta_1) - Z(\pi - \varepsilon - \theta_1)]$$

$$Z(\psi)=\frac{1}{\pi}\,\mathrm{arc\,tg}\left(\omega\,\mathrm{tg}\,\frac{\psi}{2}\right),\quad \omega=\frac{1+R_1}{1-R_1}\,.$$

To solve the problem of diffraction of a plane wave impinging on the edge of a two-sided angle formed by the wing planes (the edge of the wing), we shall use the method proposed in /3/. To find the solution inside the diffraction surface of the incident wave, we must evaluate the following integral:

$$\Phi=\Phi^{+}+\Phi^{-} \tag{2.2}$$

$$\Phi^{\pm}=\pm\frac{1}{2\pi i}\int\limits_{R^{\lambda}}^{R^{\lambda}\exp(\pm i\lambda 2\pi)} F(\cos\psi)\,\frac{(1+u\exp(\mp i\lambda\theta))\,du}{2u\,(1-u\exp(\mp i\lambda\theta))}$$

$$R=\frac{\tau}{r}-\sqrt{\frac{\tau^2}{r^2}-1},\quad r=\sqrt{x^2+y^2},\quad \theta=\mathrm{arc\,tg}\,\frac{y}{x},\quad \tau=\frac{t-z\cos\alpha}{\sin\alpha}\,.$$

Here $F(\cos\psi)=F(t,\,r,\,z,\,\cos\psi)$ is the solution behind the incident wave and $\cos\psi=\frac{1}{2}(u^{1/\lambda}+u^{-1/\lambda})$, ψ is the polar angle of the cylindrical coordinate system in which the integration is carried out.

Remembering that there is no reflected wave in the real diffraction problem, we obtain the complete solution inside the diffraction surface by evaluating integral (2.2) for the reflected wave and summing the results obtained. As a result we find that the solution inside the cone N (Fig.2) will be

$$\Phi=3+\Psi^{+}+\Psi^{-}$$

$$\Psi^{\pm}=\mp\frac{1}{\pi}\,\mathrm{arc\,tg}\,\frac{2R^{\lambda}\cos\lambda(\pi/2-\theta)\pm(1+R^{2\lambda})\cos\lambda\pi}{\sin\lambda\pi\,(1-R^{2\lambda})}\,.$$

Let us denote the solutions inside the cones A,B,C and N respectively by

$$\Phi=1-\Phi_A,\ \Phi=1-\Phi_B,\ \Phi=1+\Phi_C,\ \Phi=2+\Phi_N\,.$$

We note that there are regions in which the cones intersect. The solution in these regions will not be a simple sum of the solutions inside the intersecting cones (provided that the solution constructed is continuous at the boundary of these regions).

Thus in the region of intersection of the cones A and C (region $PQED$ in Fig.2) and of the cones A and B (region $P'Q'E'D'$ in Fig.2), we have

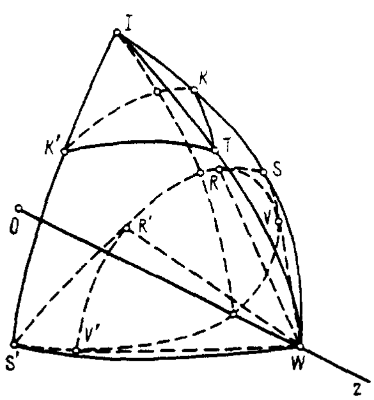
$$\Phi=1-\Phi_A-\Phi_C-L\,(PQED),$$
$$\Phi=1-\Phi_A-\Phi_B-L\,(P'Q'E'D')\,.$$

In the region of intersection of cones B, C but outside the cone N (region $IKTGK'$; the detailed form of this and the following regions is given in Fig.4) we obtain

$$\Phi=1-\Phi_B-\Phi_C$$

In the region of intersection of cones B,C and N $(S'K'KSWT)$ we find $\Phi=\Phi_B-\Phi_C-\Phi_N$

Fig.4



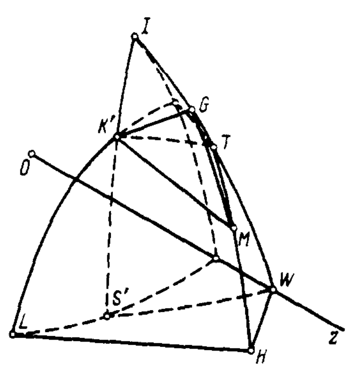


Fig.5

Inside the regions $HLK'TW$ (Fig.5) and $H'L'KTW$ (this region is symmetrical with respect to $HLK'TW$ about the plane yOz) where the cone N intersects one of the cones B or C, the solution is

$$\Phi=1+\Phi_B+\Phi_N,\ \Phi=1+\Phi_C+\Phi_N$$

In the regions situated inside one of the cones C and B behind the reflected plane wave but outside the other cone and also outside the cone N (region $GTK'M$ in Fig.5 and region

$GTKM'$ symmetrical with respect to the previous region relative to the plane yOz). the respective solutions will be

$$\Phi = 2 + \Phi_B\,(GTKM'),\quad \Phi = 2 + \Phi_C\,(GTK'M)\,.$$

Fig.4 shows that a region exists in which the conical waves B and C are diffracted at the wing edge (cone W) and regions ($WS'R'V'$ and $WSRV$) where these waves are reflected from the wing surface. Henceforth, we shall call the conical waves B and C incident cones, and their reflections from the wing surface, reflected cones. If $\Phi\,(t, r, z, \theta)$ is a solution behind the incident cone written in the cylindrical coordinate system obtained from the initial x, y, z coordinates, then the solution behind the reflected cone will be given by the formula

$$\Phi = \Phi\,(t, r, z, \theta) + \Phi\,(t, r, z, \pi \pm 2\beta - \theta)$$

(the upper and lower signs correspond to cones B and C respectively).

Let us put (Φ_{B1} and Φ_{C1} are the complete solutions behind the cones B and C)

$$\Phi_{B1}\,(t, r, z, \pi + 2\beta - \theta) = 1 + \Phi_{B0},\quad \Phi_{C1}\,(t, r, z, \pi - 2\beta - \theta) = 1 + \Phi_{C0}\,.$$

The solution inside the cone W is also found using the method given in /3/. Using integral (2.2) we obtain

$$\Phi_W = \sum_{j=1}^{2}\sum_{m=1}^{2}(Y^{+}_{mj} + Y^{-}_{mj})$$

$$Y^{\pm}_{mj} = \mp\frac{1}{2\pi i}\int_{R^{\lambda}}^{R^{\lambda}\exp(\pm i\lambda\pi)} F_m \frac{(1 + u\exp(\mp i\lambda\theta_{jm}))\,du}{2u\,(1 - u\exp(\mp i\lambda\theta_{jm}))}$$

$$F_1 = \Phi_B\,(t, r, z, \psi) - 1,\quad F_2 = \Phi_C\,(t, r, z, \psi) - 1$$

$$\cos\psi = {}^1/_2\,(u^{1/\lambda} + u^{-1/\lambda}),\quad R = \eta - (\eta^2 - 1)^{1/2}$$

$$\eta = \frac{t - z\zeta_0\sin\mu - (\zeta_0^2 - 1)^{1/2}(z^2 - r^2 - t^2)^{1/2}}{\zeta_0 r\sin\mu},\quad \lambda = \frac{\pi}{\pi - 2\beta}$$

$$\theta_{11} = \theta - \frac{\pi}{2} + \beta,\quad \theta_{12} = \frac{\pi}{2} + 3\beta - \theta,\quad \theta_{21} = \theta + \frac{\pi}{2} - \beta,$$

$$\theta_{22} = \frac{3\pi}{2} - 3\beta - \theta$$

where ζ_0, μ are the same as in (2.1).

It should be noted that the reflected cones and the cone W lie inside the intersection of the cones B, C and N. Therefore the complete solution in these regions is

$$\Phi = \Phi_{B0} + \Phi_B + \Phi_C + \Phi_N - 1\quad (WS'R'V')$$
$$\Phi = \Phi_{C0} + \Phi_B + \Phi_C + \Phi_N - 1\ (WSRV)$$
$$\Phi = \Phi_W + \Phi_N\ (WV'R'RV)\,.$$

Having found the solutions in all regions adjacent to the sphere, we obtain the solution inside the sphere from (1.2). Here, thanks to the condition $\partial\Phi/\partial n = 0$, the boundary conditions on the sphere readjust themselves on the wing surface so that the symmetry about the planes $\theta = \beta$, $\theta = \pi - \beta$ (wing planes) is maintained. The integration is carried out over the region

$$2\beta - \frac{\pi}{2} < \theta < \frac{3\pi}{2} - 2\beta,\quad 0 < \varphi < \pi;\quad T = 2\pi - 4\beta\,.$$

The solution obtained serves to describe the flow on the top side of the wing. A solution for the bottom side is sought as follows: the relation $\Phi_+ + \Phi_- = 2$ holds in regions not adjacent to the wing edge (here Φ_+ and Φ_- are the solutions on the top and bottom side of the wing). In the regions of diffraction on the wing edge the solution is found in the same manner as in the corresponding regions on the top side of the wing, but the period of the solution in θ will be $T = 2\pi + 4\beta$ and $\lambda = \pi/(\pi - 2\beta)$ will be replaced by $\lambda_1 = \pi/(\pi - 2\beta)$. The same period (and the same λ_1) will appear in formula (1.2) in the course of computing the solution inside the diffraction sphere on the bottom side of the wing.

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Translated by L.K.

PMM U.S.S.R.,Vol.49,No.2,pp.195-199,1985
Printed in Great Britain

0021-8928/85 $10.00+0.00
Pergamon Journals Ltd.

THE CAUCHY PROBLEM FOR A QUASILINEAR SYSTEM WHEN THERE ARE CHARACTERISTIC POINTS ON THE INITIAL SURFACE*

V.A. KULIKOVSKII

The problem of the existence, uniqueness and analyticity of a solution of the Cauchy problem in complex and real spaces for a quasilinear analytical set of equations are examined, when the initial data are specified on an analytical surface containing characteristic points, and an error occurs in the initial data and set of equations. In particular, the Cauchy problem with initial data on the envelope of one of the families of the characteristic surfaces of the system is examined.

Discontinuities, whose trajectories are envelopes of the characteristic surfaces, are encountered when studying Chapman-Zhug detonation waves in gas dynamics /1—3/ and magneto-hydrodynamics /4, 5/, and also in the theory of avalanches /6/. The construction of solutions around envelopes of the characteristic surfaces is interesting both in connection with the new problems of detonation in gases - taking into account the inhomogeneity of the background, intakes of mass, momentum and energy to the gas and distortion of the wave front - and in connection with other models.

Investigations of similar problems have so far been confined to linear systems /7—12/, whose knowledge of the order of contact of the characteristic surfaces and initial manifold was substantially used.

1. Consider the set of first-order quasilinear equations in the m-dimensional complex space $x_1, \ldots, x_m$ whose coefficients and right-hand sides are complex functions analytic in the variables $x_1, \ldots, x_m, u_1, \ldots, u_n$

$$\sum_{k=1}^{m}\sum_{j=1}^{n} a_{ijk}\frac{\partial u_j}{\partial x_k} + b_i = 0, \quad i = 1, \ldots, n\ . \tag{1.1}$$

Suppose the analytical initial values of the unknown functions are given on the analytical surface S of complex codimensionality 1, such that the surface S is an envelope of one of the families of the characteristic surfaces of (1.1). We can assume, without loss of generality, that $u_i|_S = 0\ (i = 1, \ldots, n)$ and in some domain D the surface S is specified by the relation $x_1 = 0$. The well-known conditions of non-solvability (1.1) relative to $\partial u_i/\partial x_1\ (i = 1, \ldots, n)$: $\operatorname{rank}\{a_{ij1}\} = n - 1$, $\operatorname{rank}\{a_{ij1} \mid b_i\} = n$ hold on the surface $S: x_1 = 0$. The latter condition can be written in the form

$$\begin{vmatrix} b_1 & a_{121} & a_{131} & \ldots & a_{1n1} \\ b_2 & a_{221} & a_{231} & \ldots & a_{2n1} \\ \vdots & \vdots & \vdots & & \vdots \\ b_n & a_{n21} & a_{n31} & \ldots & a_{nn1} \end{vmatrix} \neq 0\ . \tag{1.2}$$

Hence it follows that there is no classical solution to the Cauchy problem with initial data on the envelope of characteristic surfaces.

We shall investigate the problem of the existence of the continuous functions $u_i\ (i = 1, \ldots, n)$, which satisfy the initial conditions on S and (1.1) outside S.

*Prikl.Matem.Mekhan.,49,2,258-264,1985